Problem 6a

$$\lim_{h \to 0} \frac{(x+h)^2 - x^2}{h}$$

since $f(x) = \frac{(x+h)^2 - x^2}{h}$

and $f(x) = \frac{x^2 - x^2}{0}$ when $h \to 0$

$= \frac{0}{0}$ ← indeterminate form

we need to DO some Algebra to find limit

$(x+h)^2 = (x+h)(x+h)$

$= x^2 + hx + hx + h^2$

$= x^2 + 2xh + h^2$

So $f(x) = \frac{\cancel{x^2} + 2xh + h^2 - \cancel{x^2}}{h}$

$= \frac{2xh + h^2}{h}$

$\boxed{f(x) = \frac{h}{h}(2x+h)} \Rightarrow f(x) = 2x+h$

Problem (1a) cont

So $\lim_{h \to 0} f(x) \longrightarrow \lim_{h \to 0} 2x+h$

since $h \to 0$ $\lim_{h \to 0} f(x) = 2x+0$
$= 2x$

the particular function that this type of limit is the definition of the derivative of is $y = x^2$

Answer 1a $= 2x$

Answer 1e

Answer 1h

Problem (1b)

$$\lim_{x \to 2} \frac{5x^3 - 20x}{2x-4}$$

Since $x \to 2$ & $f(x) = \frac{5x^3-20x}{2x-4}$

$$f(2) = \frac{5(2)^3 - 20(2)}{2(2)-4}$$

$$= \frac{40-40}{4-4} = \frac{0}{0}$$

Indeterminate Form

DO SOME ALGEBRA!

$$f(x) = \frac{5x^3-20x}{2x-4} = \frac{5x(x^2-4)}{2(x-2)}$$

$$= \frac{5x\cancel{(x-2)}(x+2)}{2\cancel{(x-2)}} = \frac{5x(x+2)}{2}$$

we just REMOVED a discontinuity at $x=2$

(1b) Since $f(x) = \frac{5x(x+2)}{2}$ with $x \neq 2$

$\lim_{x \to 2} f(x) = \frac{5(2)(2+2)}{2} = \frac{10(4)}{2} = \frac{40}{2}$

$= 20$

So $\boxed{\lim_{x \to 2} \frac{5x^3 - 20x}{2x-4} = 20}$

(1b)

Also (1i) $\rightarrow$ this has a Removable Discontinuity at $x=2$

It is $(2, 20)$ $\rightarrow$ hole in $f(x)$

Problem 1c

$$\lim_{x \to 2} \frac{5x^3+24x}{2x+4}$$

$$f(x) = \frac{5x^3+24x}{2x+4} \qquad f(2) = \frac{5(2)^3+24(2)}{2(2)+4}$$

$$= \frac{40+48}{8}$$

$$= \frac{88}{8} = 11$$

$$\boxed{\lim_{x \to 2} \frac{5x^3+24x}{2x+4} = 11}$$

1F

Problem 1e

This is the only limit that can be found using direct substitution!!

Problem 1b

$$\lim_{x\to 2} \frac{5x^3+24x}{2x-4}$$

$$f(x) = \frac{5x^3+24x}{2x-4}$$

$$f(2) = \frac{5(2)^3+24(2)}{2(2)-4} = \frac{40+48}{4-4}$$

$$= \frac{88}{0} = \text{undefined}$$

So $\boxed{\lim_{x\to 2} \frac{5x^3+24x}{2x-4} = DNE}$ b

Problem 1g

this is a function without a limit at $x=2$

(2) $\lim_{x \to 12} f(x) = 2$

$\lim_{x \to 12} g(x) = 6$

$\lim_{x \to 12} h(x) = 9$

$$\boxed{\text{Now I Am a math teacher so my proofs will be more involved than yours}}$$

$$\lim_{x \to 12} \frac{f(x) - 2g(x)}{7 + h(x)f(x)} = \lim_{x \to 2} \frac{n(x)}{d(x)}$$

with $\left.\begin{aligned} n(x) &= f(x) - 2g(x) \\ d(x) &= 7 + h(x)f(x) \end{aligned}\right\}$ this simplifies ~~proof~~ proFess

② cont

$$\lim_{x\to 12} \frac{n(x)}{d(x)} = \frac{\lim_{x\to 12} n(x)}{\lim_{x\to 12} d(x)}$$ (By Limit of a Quotient)

$$\lim_{x\to 12} n(x) = \lim_{x\to 12} f(x) - 2\lim_{x\to 12} g(x)$$

(by limit of a difference & constant multiplier)

So $\lim_{x\to 12} n(x) = 2 - 2(6)$
$= 2 - 12 = -10$

(by substitution)

$$\lim_{x\to 12} d(x) = 7 + \lim_{x\to 12} h(x) \cdot \lim_{x\to 12} f(x)$$

(By limit of constant and limit of a product)

So $\lim_{x\to 12} d(x) = 7 + (9)(2) = 7 + 18$
$= 25$

(2) cont

$$\lim_{x \to 12} \frac{f(x) - 2g(x)}{7 + h(x)f(x)} = \frac{-10}{25}$$

Therefore $= \frac{-2}{5} = -0.4$

③ $\lim_{x \to -1} f(x) = 0$

$\lim_{x \to -1} g(x) = 9$

$\lim_{x \to -1} h(x) = -7$

$$\lim_{x \to -1} \sqrt[4]{\frac{2+g(x)}{1-10h(x)}}$$

let $\frac{2+g(x)}{1-10h(x)} = R(x)$

So $\lim_{x \to -1} \sqrt[4]{R(x)} = \sqrt[4]{\lim_{x \to -1} R(x)}$

(Limit of a Power) if $R(x) > 0$ for all x

Problem ③ cont

So find limit of $R(x)$ at $x=-1$

$$\lim_{x\to -1} R(x) = \lim_{x\to -1}\left(\frac{n(x)}{d(x)}\right)$$

$$= \frac{\lim_{x\to -1} n(x)}{\lim_{x\to -1} d(x)}$$ with $n(x) = 2+g(x)$

$d(x) = 1 - 10h(x)$

(Limit of Quotient)

So $\lim_{x\to -1} n(x) = \lim_{x\to -1}(2+g(x))$

$= 2 + \lim_{x\to -1} g(x)$

by limit of a sum & limit of a constant $= 2 + 9$

$= 11$

③ cont

So $\lim_{x \to -1} d(x) = \lim_{x \to -1} (1 - 10h(x))$

& $= 1 - 10 \lim_{x \to -1} h(x)$ (limit of a constant & constant multiplier & limit of a difference

So $\lim_{x \to -1} d(x) = 1 - 10(-7)$

$= 1 + 70$

$= 71$

So $\lim_{x \to -1} \sqrt[4]{\frac{2-g(x)}{1-10h(x)}} = \sqrt[4]{\frac{4}{71}}$